

Exercise 2

Explain what it means to say that

$$\lim_{x \rightarrow 1^-} f(x) = 3 \quad \text{and} \quad \lim_{x \rightarrow 1^+} f(x) = 7$$

In this situation is it possible that $\lim_{x \rightarrow 1} f(x)$ exists? Explain.

Solution

$$\lim_{x \rightarrow 1^-} f(x) = 3$$

means that as x approaches 1 from below ($0.9, 0.99, \dots$), the function approaches the value 3. On the other hand,

$$\lim_{x \rightarrow 1^+} f(x) = 7$$

means that as x approaches 1 from above ($1.1, 1.01, \dots$), the function approaches the value 7. Since the function has different limits as x approaches 1, $\lim_{x \rightarrow 1} f(x)$ does not exist.